# Quantum computation with trapped electrons

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# Why trapped electrons?

- Lighter mass brings to higher trapping frequencies (higher clock speed);
- Microwave and radiofrequency radiation;
- Extremely good isolation from the environment (negligible damping and reduced thermal fluctuations);
- High precision spectroscopy and measurements (*g* factor);
- Quantized external (motional) and internal (spin) degrees of freedom;
- Ground state cooling of cyclotron motion.

# Outline

- Single electron in a Penning trap
- Qubit encoding and manipulation
- Scalability
- Coupling qubits
- How to engineer relevant quantum spin systems (Ising, XY models …)
- Conclusions and future perspectives

# How Penning traps work

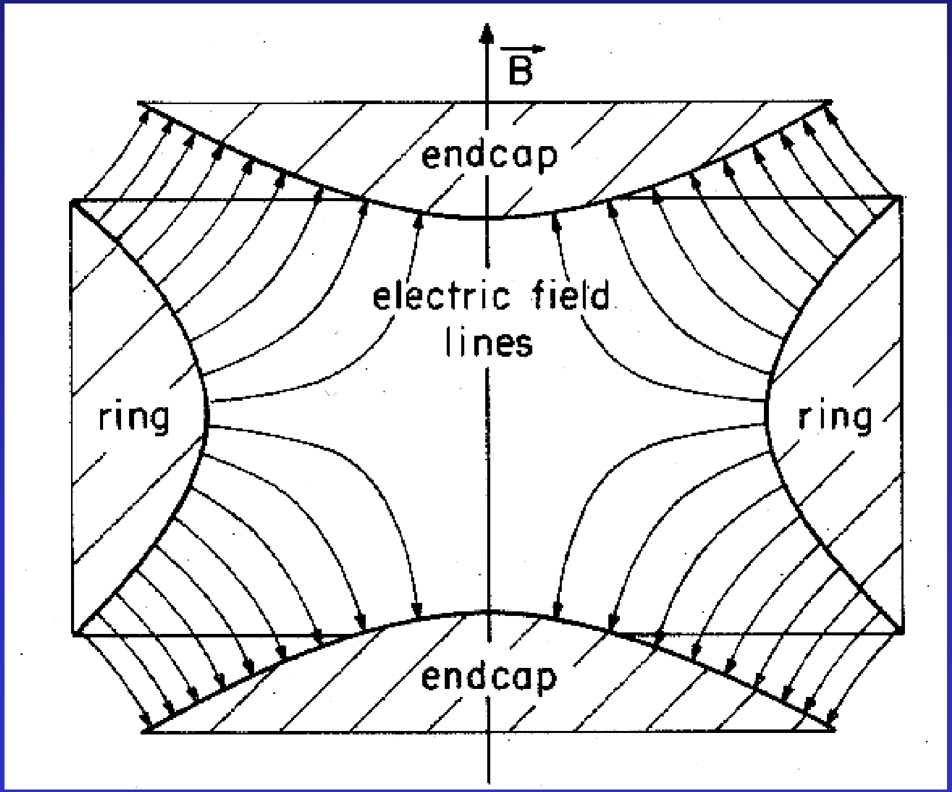
L. S. Brown and G. Gabrielse, Rev. Mod. Phys. **58**, 233 (1986)
F. G. Major, V. Gheorghe, and G. Werth, *Charged particle traps* (Springer, Heidelberg, 2005)

**STATIC electric and magnetic FIELDS:**

**Homogeneous magnetic field** $\rightarrow$ radial confinement (cyclotron motion)

**Quadrupole potential** $\rightarrow$ axial confinement and slow circular drift in the radial plane (magnetron motion)

$$V(x,y,z) = V_0 \frac{2z^2 - (x^2 + y^2)}{2d^2}$$



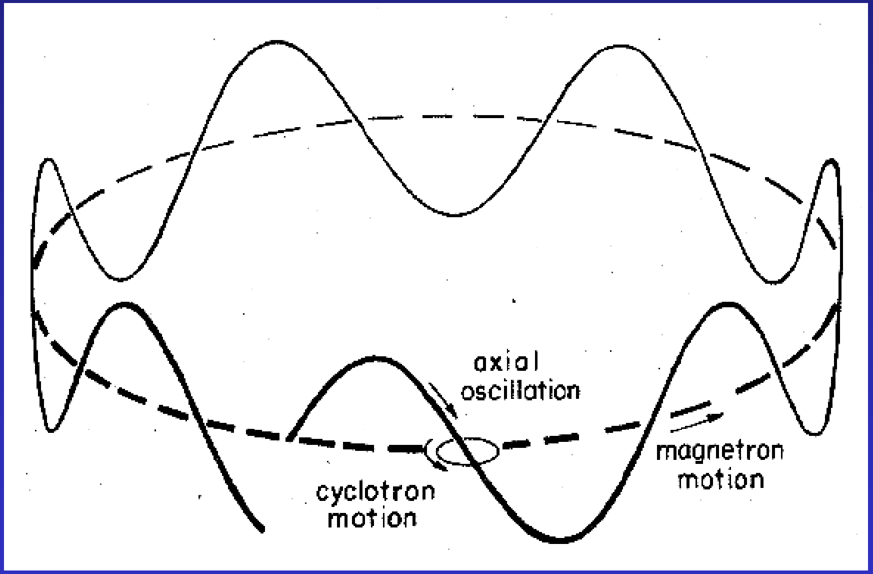
# Electron motion inside the trap

For a trap size of 1 cm, a voltage $V_0$ ~ 10 V, a magnetic field $B$ ~ 3 T

$$\frac{\omega_c}{2\pi} \approx \frac{1}{2\pi}\frac{|e|B}{m} \approx 100\,\mathrm{GHz}$$

$$\frac{\omega_z}{2\pi} \equiv \frac{1}{2\pi}\sqrt{\frac{eV_0}{md^2}} \approx 30\,\mathrm{MHz}$$

$$\frac{\omega_m}{2\pi} \approx \frac{1}{2\pi}\frac{\omega_z^2}{2\omega_c} \approx 5\,\mathrm{kHz}$$



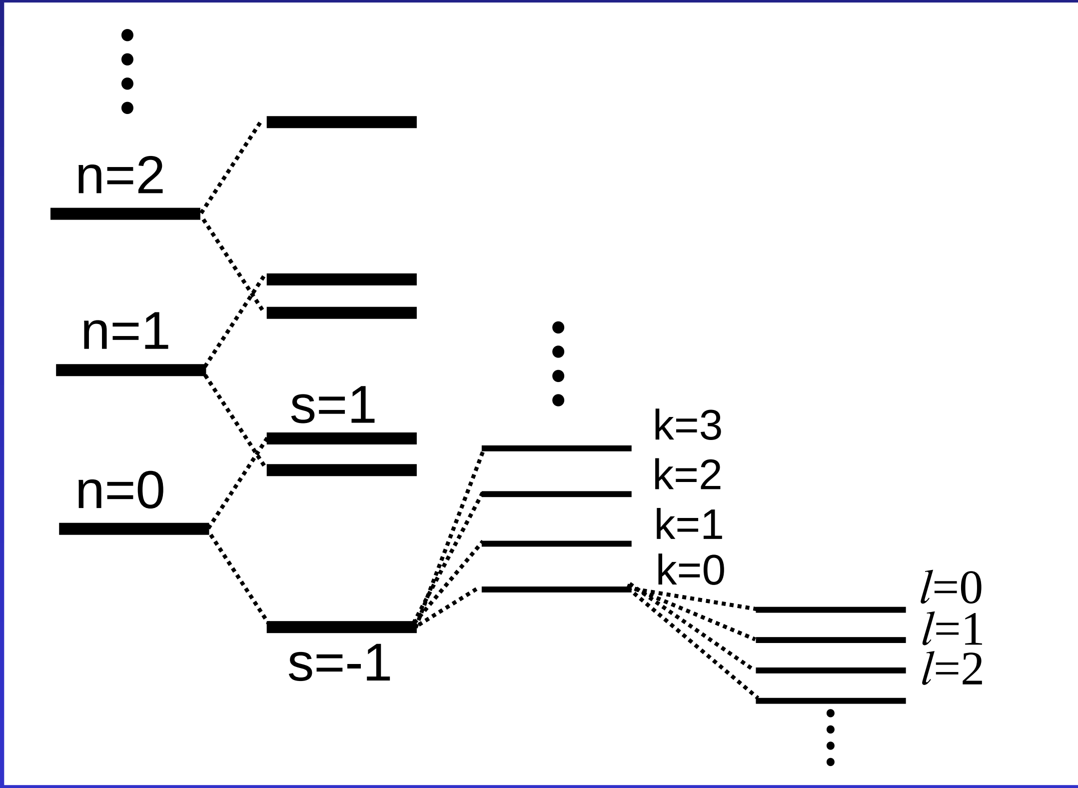
# Energy level structure

$$E = -\hbar\omega_m\left(l+\frac{1}{2}\right)+\hbar\omega_z\left(k+\frac{1}{2}\right)+\hbar\omega_c\left(n+\frac{1}{2}\right)+\frac{\hbar\omega_s}{2}s$$

with $l,k,n = 0,1,2,\ldots$

and $s=\pm 1$

$$\omega_s \equiv \frac{g}{2}\frac{|e|B}{m}$$



# Overview on damping mechanisms

Cyclotron motion: radiative decay influenced by cavity effects

$$\gamma_c \approx \frac{e^2\omega_c^2}{3\pi\varepsilon_0 mc^3} \approx 10\ \mathrm{s}^{-1}$$

Negligible thermal excitation below 4 K

Axial motion: negligible radiative damping
Dissipation due to electric field fluctuations in the external circuitry and trap components

Magnetron motion: no radiative decay, affected by device imperfections

Electron spin: coupling to the environment via its magnetic moment
➡ relaxation is negligible

VOLUME 83, NUMBER 7 PHYSICAL REVIEW LETTERS 16 AUGUST 1999

# Observing the Quantum Limit of an Electron Cyclotron: QND Measurements of Quantum Jumps between Fock States

S. Peil and G. Gabrielse
*Department of Physics, Harvard University, Cambridge, Massachusetts 02138*
(Received 18 March 1999)

Quantum jumps between Fock states of a one-electron oscillator reveal the quantum limit of a cyclotron. With a surrounding cavity inhibiting synchrotron radiation 140-fold, the jumps show a 13 s Fock state lifetime and a cyclotron in thermal equilibrium with 1.6 to 4.2 K blackbody photons. These disappear by 80 mK, a temperature 50 times lower than previously achieved with an isolated elementary particle. The cyclotron stays in its ground state until a resonant photon is injected. A quantum cyclotron offers a new route to measuring the electron magnetic moment and the fine structure constant.

PACS numbers: 03.65.–w, 42.50.Ct

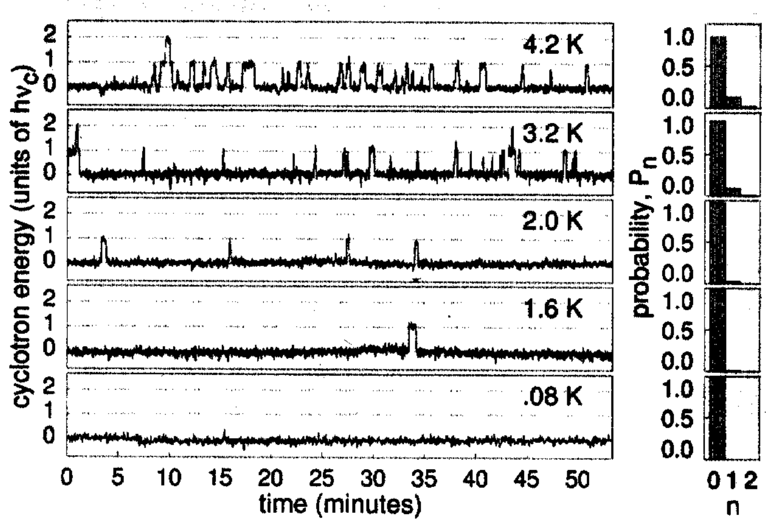



FIG. 2. Quantum jumps between the lowest states of the one-electron cyclotron oscillator decrease in frequency as the cavity temperature is lowered.

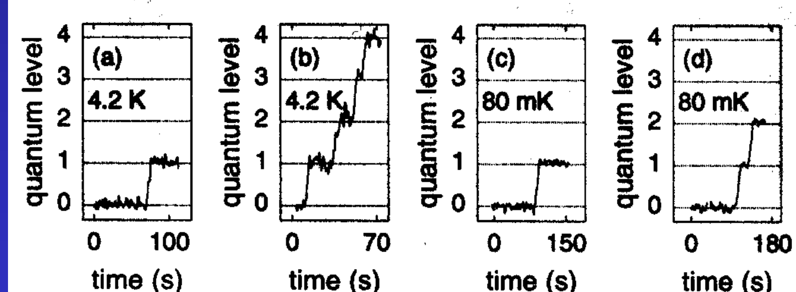



FIG. 3. Excitations to excited Fock states which are stimulated by 4.2 K blackbody photons in (a) and (b), and by an externally applied microwave field in (c) and (d).

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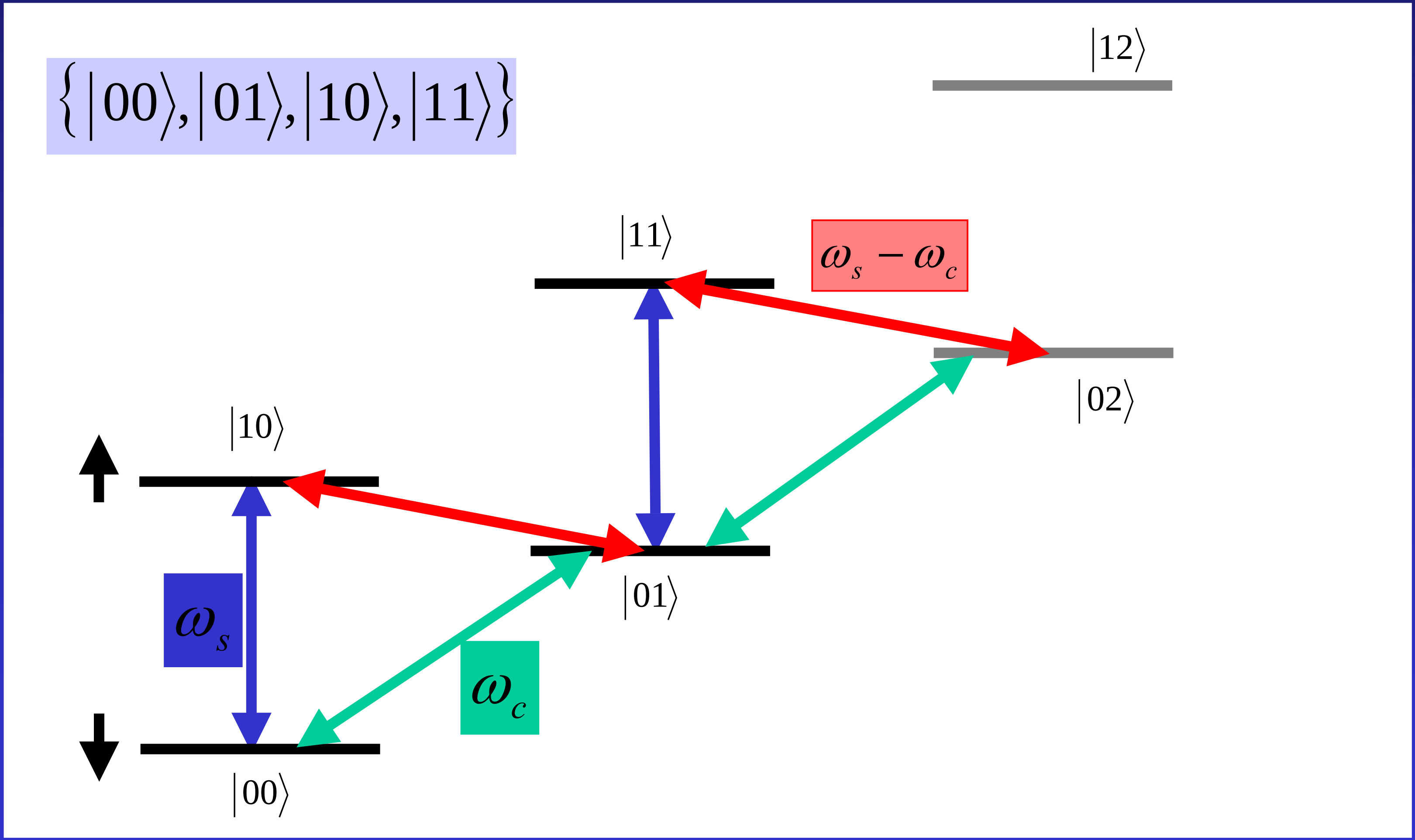
# Storing the qubits in the electron spin and cyclotron motion

S. Stortini and I. M., Eur. Phys. J. D **32**, 209 (2005)



# Single qubit operations (1)

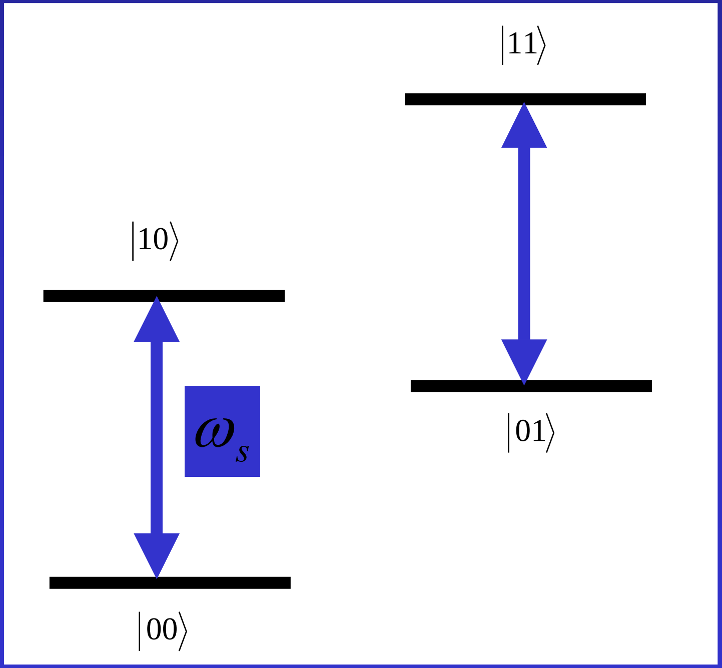
Spin: resonant pulses at the spin transition frequency $\omega_s$

$$\mathbf{b}(t) = b_0\left[\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}\right]$$

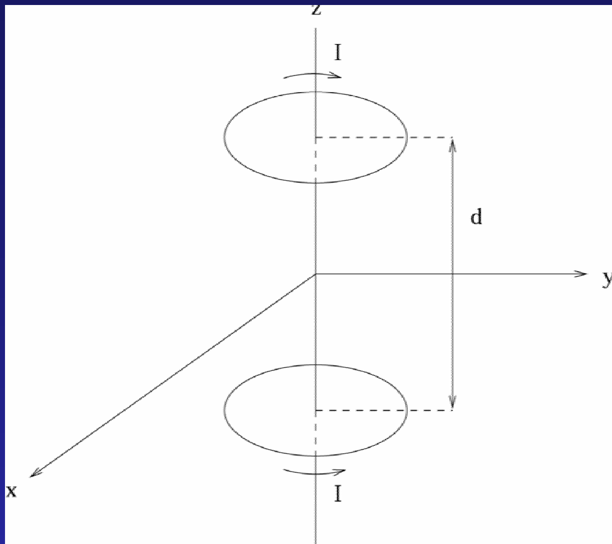
$$H_{\text{int}} = -\boldsymbol{\mu}\cdot\mathbf{b}(t) = \frac{g}{2}\mu_B \boldsymbol{\sigma}\cdot\mathbf{b}(t) = \frac{\hbar\Omega_s}{2}(\sigma_x \cos\omega t + \sigma_y \sin\omega t)$$

$$|0\rangle = |0\rangle\cos\left(\frac{\Omega_s t}{2}\right) - i|1\rangle\sin\left(\frac{\Omega_s t}{2}\right)$$

$$|1\rangle = |1\rangle\cos\left(\frac{\Omega_s t}{2}\right) - i|0\rangle\sin\left(\frac{\Omega_s t}{2}\right)$$



# Cyclotron qubit manipulation




$$I(t) = I\cos(\omega_d t + \phi) \longrightarrow \mathbf{b}(t) = b_1 \boldsymbol{\rho}(t)\cos(\omega_d t + \phi)$$

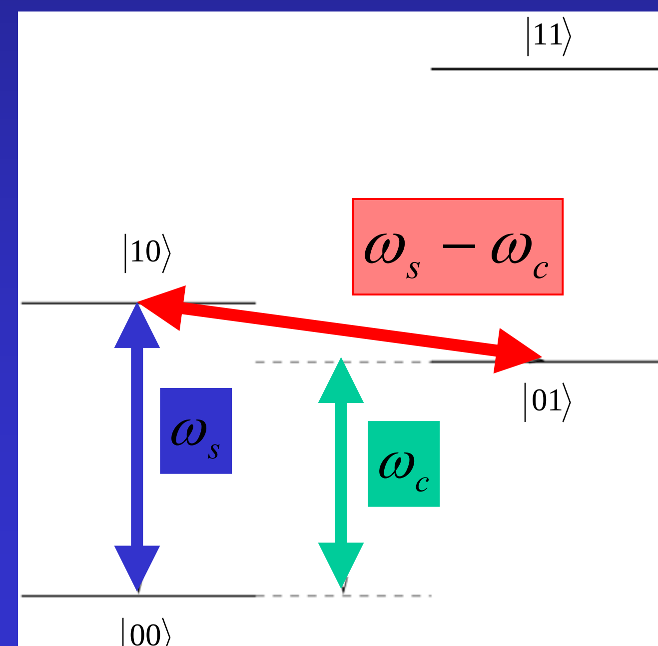
$$b_1 = \frac{I\pi a^2 3d}{c\left(a^2 + d^2/4\right)^{5/2}}$$

$$H_{\text{int}} = -\frac{g\mu_B}{2}\left[\sigma_x x(t) + \sigma_y y(t)\right] b_1 \cos(\omega_d t + \phi)$$

in RWA

$$H_{\text{int}}^{(IP)} \approx -\frac{g\mu_B b_1}{2}\sqrt{\frac{\hbar}{2m\tilde{\omega}_c}}\left(\sigma_+ a_c e^{-i\phi} + \sigma_- a_c^+ e^{i\phi}\right)$$

$$\tilde{\omega}_c \equiv \sqrt{\omega_c^2 - 2\omega_z^2}$$



# Unitary time evolution

$$U(t) = \exp\left(-\frac{i}{\hbar} H_{\text{int}}^{(IP)} t\right) \quad \Longrightarrow \quad U(t) = C(t) + iS(t)$$

$$C(t) = \sigma_+\sigma_- \cos\left(\lambda\sqrt{a_c a_c^+}\right) + \sigma_-\sigma_+ \cos\left(\lambda\sqrt{a_c^+ a_c}\right)$$

$$S(t) = \sigma_+ e^{-i\phi} \frac{\text{sen}\left(\lambda\sqrt{a_c a_c^+}\right)}{\sqrt{a_c a_c^+}} a_c + \sigma_- e^{i\phi} \frac{\text{sen}\left(\lambda\sqrt{a_c^+ a_c}\right)}{\sqrt{a_c^+ a_c}} a_c^+$$

$$\lambda = \frac{\theta}{2}$$

$$\theta = g\mu_B b_1 t \sqrt{\frac{1}{2m\hbar\tilde{\omega}_c}}$$

$$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle, |02\rangle\}$$

$$M(\theta,\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\left(\frac{\theta}{2}\right) & i\,\text{sen}\left(\frac{\theta}{2}\right)e^{i\phi} & 0 & 0 \\ 0 & i\,\text{sen}\left(\frac{\theta}{2}\right)e^{-i\phi} & \cos\left(\frac{\theta}{2}\right) & 0 & 0 \\ 0 & 0 & 0 & \cos\left(\frac{\theta}{2}\sqrt{2}\right) & ie^{-i\phi}\,\text{sen}\left(\frac{\theta}{2}\sqrt{2}\right) \\ 0 & 0 & 0 & ie^{i\phi}\,\text{sen}\left(\frac{\theta}{2}\sqrt{2}\right) & \cos\left(\frac{\theta}{2}\sqrt{2}\right) \end{pmatrix}$$

# Single qubit operations (2)

## Composite pulses technique

A. M. Childs and I. L. Chuang, PRA **63**, 012306 (2001);

S. Gulde *et al.*, Nature **421**, 48 (2003)

Cyclotron: 1) Swap the cyclotron and spin qubits

$$M\left(\frac{\pi}{\sqrt{2}},0\right) \quad M\left(\frac{2\pi}{\sqrt{2}},\phi_s\right) \quad M\left(\frac{\pi}{\sqrt{2}},0\right) \qquad SWAPPING$$

$$\phi_s = \arccos\left[\cot^2\left(\frac{\pi}{\sqrt{2}}\right)\right]$$

2) Operate on the spin qubit

3) Swap back the cyclotron and spin qubits

$$M\left(\frac{\pi}{\sqrt{2}},\pi\right) \quad M\left(\frac{2\pi}{\sqrt{2}},\pi+\phi_s\right) \quad M\left(\frac{\pi}{\sqrt{2}},\pi\right) \qquad (SWAPPING)^{-1}$$

# Two-qubit gate: Conditional phase shift

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \xrightarrow{\text{C-PS}} \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \mathrm{e}^{\mathrm{i}\phi}\,\delta|11\rangle$$

$$C - PS(\phi = \pi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$M(\pi,0) \times M\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{2}\right) \times M(\pi,0) \times M\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

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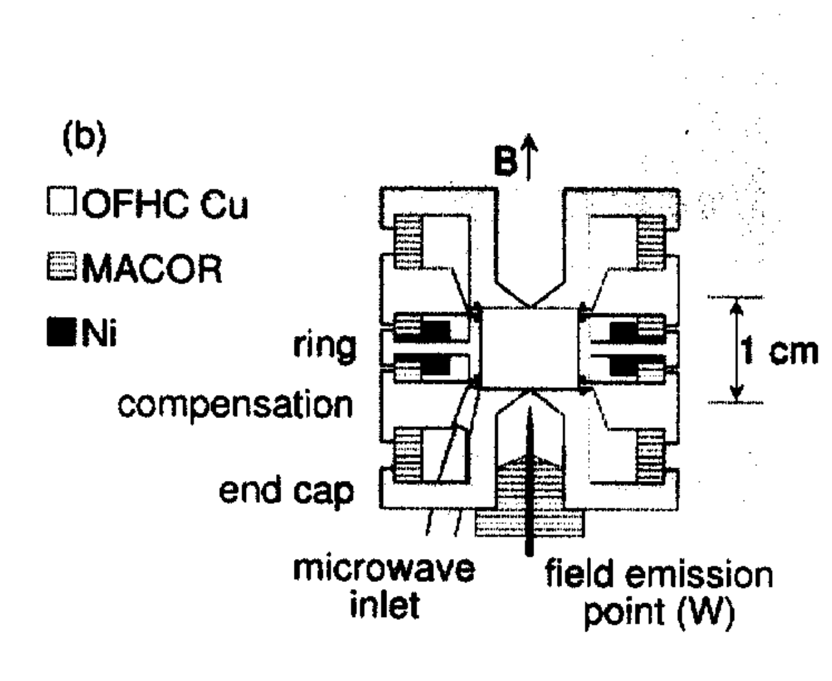
# From a cylindrical Penning trap …

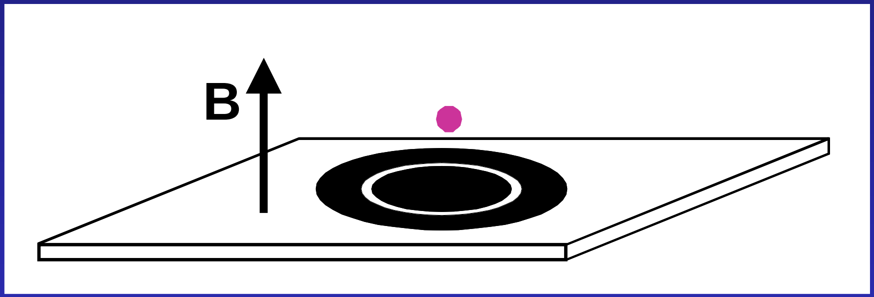
S. Peil and G. Gabrielse, Phys. Rev. Lett. **83**, 1287 (1999)

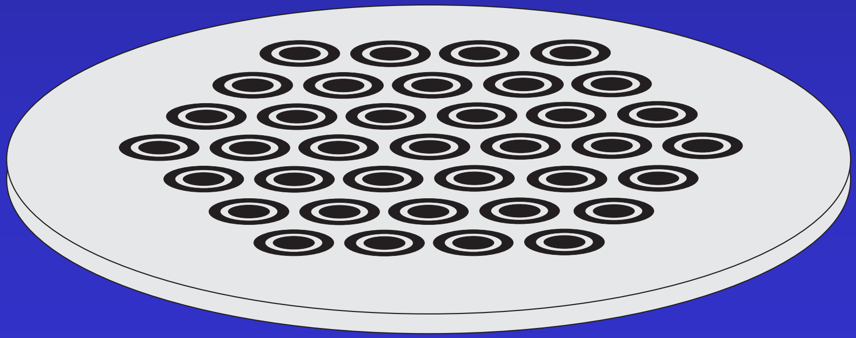
- Microwave cavity [J. N. Tan & G. Gabrielse, PRL **67**, 3090 (1991)];
- Well defined cavity modes;
- Quality factor $Q = 5 \times 10^4$
- Cavity-induced suppression of spontaneous emission of synchrotron radiation.



# … to a planar Penning trap

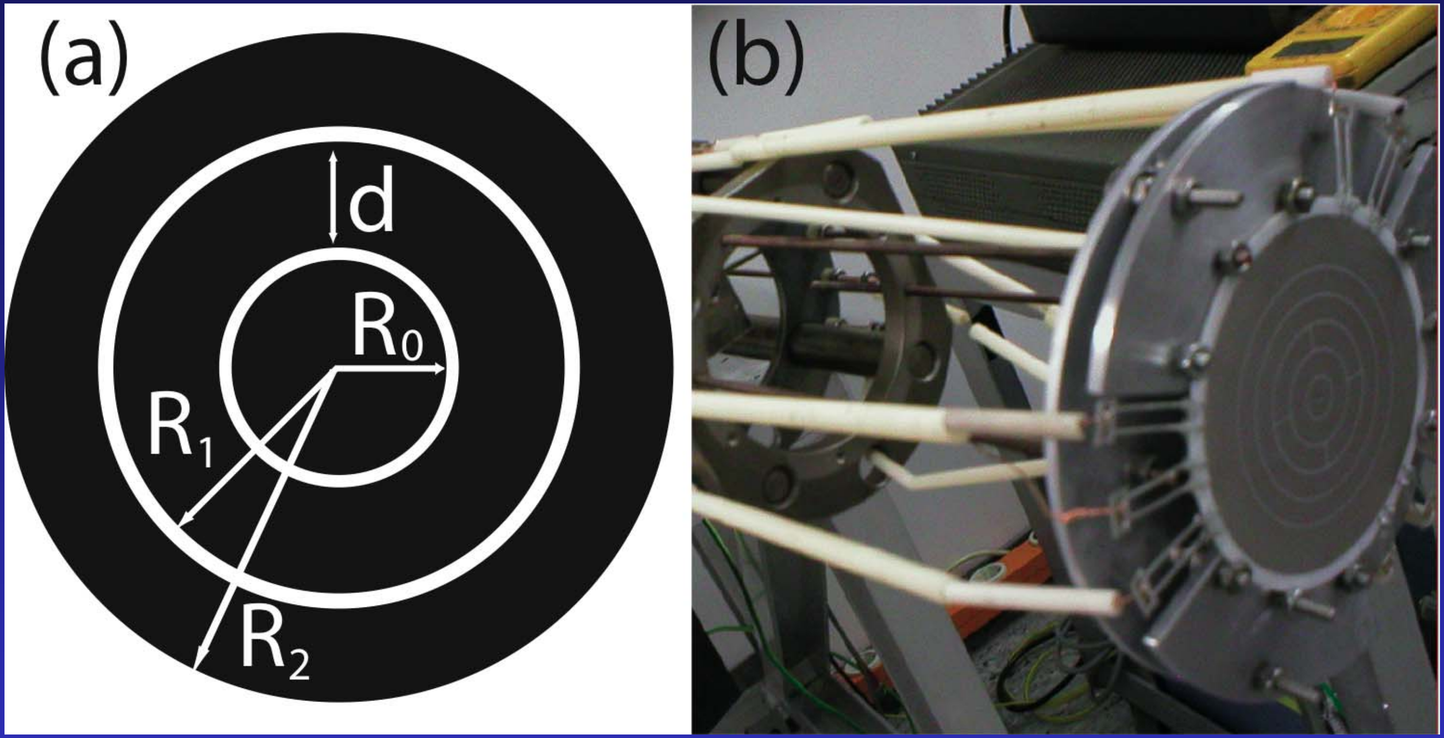
S. Stahl, F. Galve, J. Alonso, S. Djekic, W. Quint, T. Valenzuela, J. Verdú, M. Vogel, and G. Werth, Eur. Phys. J. D **32**, 139 (2005)



Electrodes printed on an insulating substrate ($Al_2O_3$):

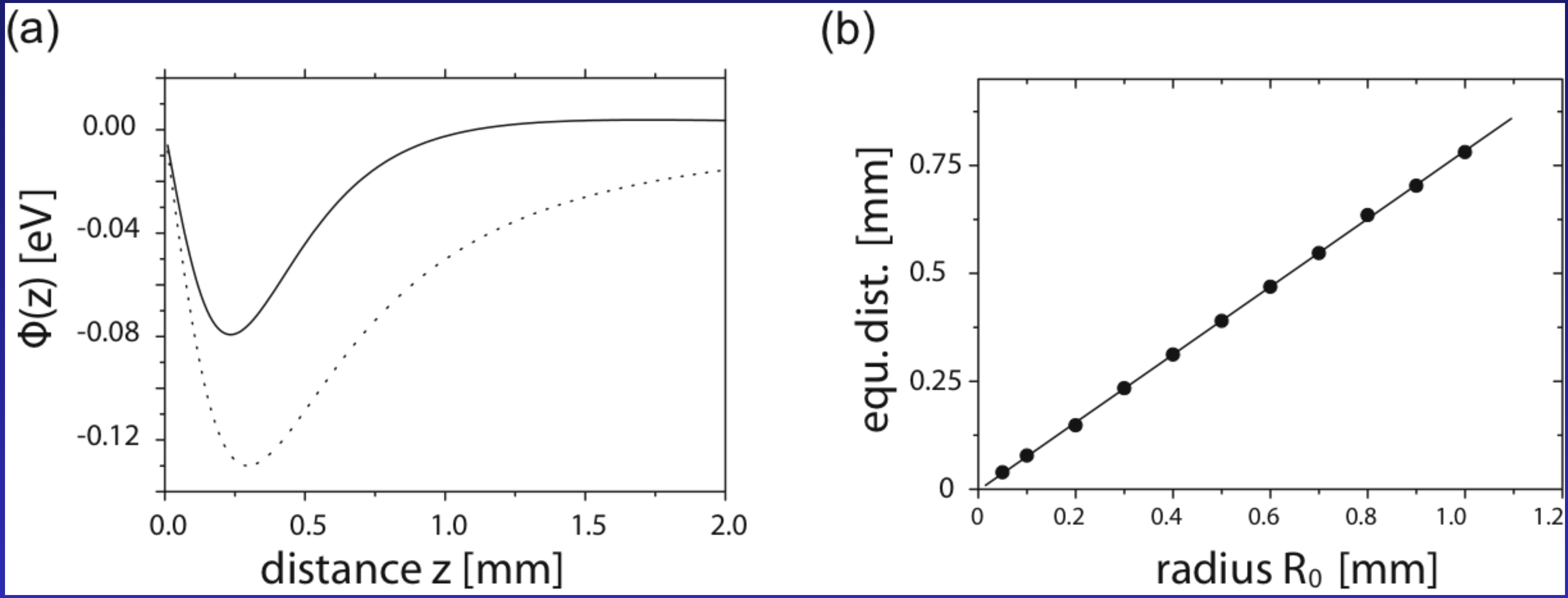
- easily produced and miniaturized (thick- and thin-film technology)
- open geometry for easy access with radiation
- more traps on the same substrate to form a two-dimensional array




Silver plated $Al_2O_3$ ceramic disk with electrodes of

$R_0$=2.5 mm, $R_1$=5.8 mm, $R_2$=9.1 mm and d=3 mm

# Axial potential



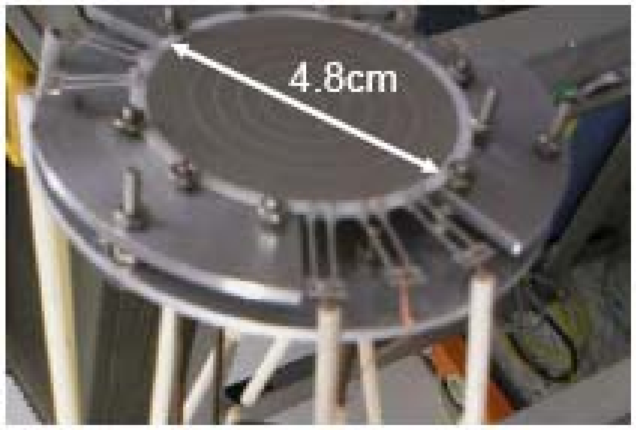

$R_0$=300 μm, $R_1$=600 μm, and $R_2$=900 μm

$U_0$ =0 V, $U_1$ =0.5 V, $U_2$ =0 V (compensation voltage –0.417 V)

$\omega_z/(2\pi)$ = 89.9 MHz (99.0 MHz solid line)

# Operation of *mm*-sized planar traps

F. Galve and G. Werth, Proc. 2006 Non-Neutral Plasma Workshop, Åarhus (2006);
F. Galve, PhD thesis, Johannes-Gutenberg-Universität Mainz (2006)




Planar trap with D = 4.8 cm,
operated at room temperature

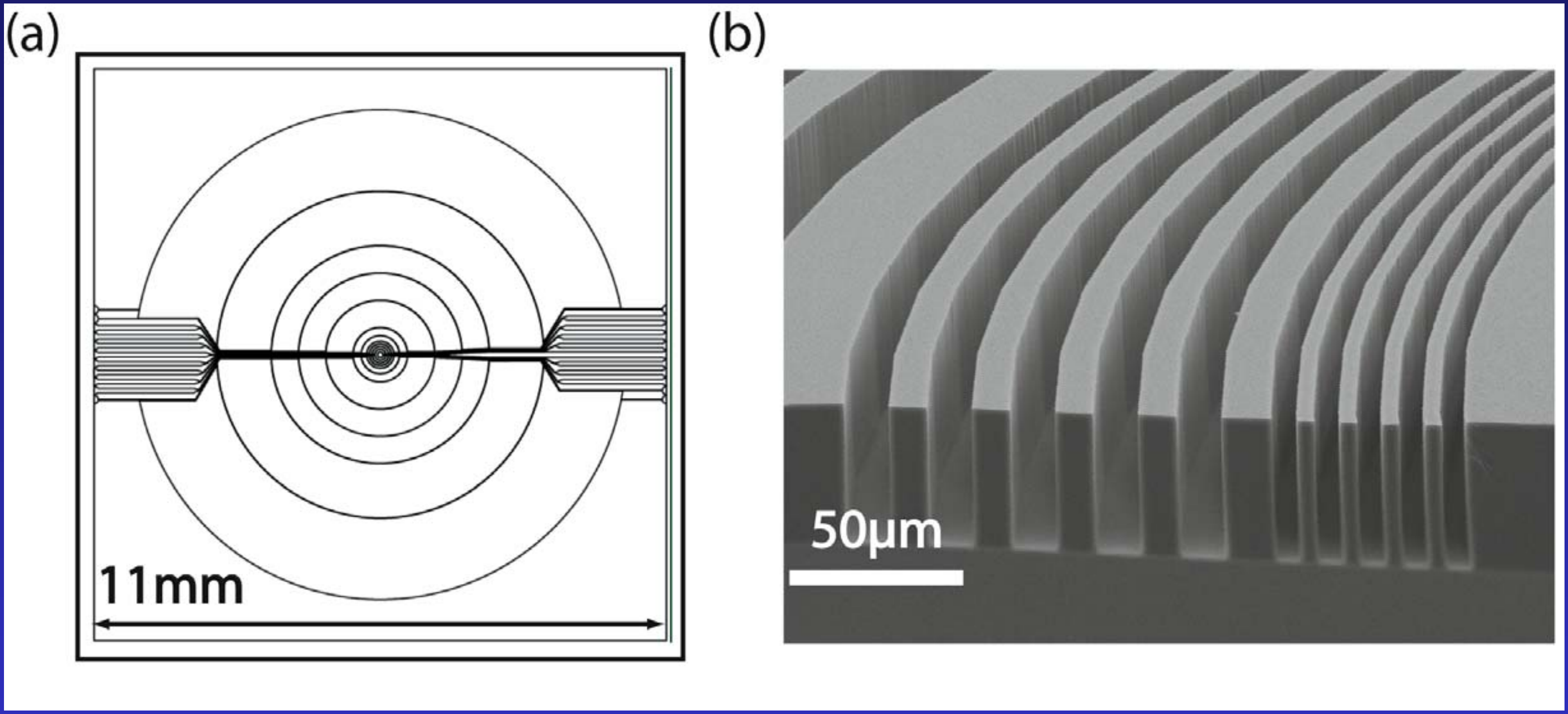
Optimized parameters:
$U_0$=0 V, $U_1$= -13.6 V, $U_2$=33.6 V
$\omega_z/2\pi$ ~ 35 MHz

Planar trap with D = 2.0 cm,
$U_0$=0 V, $U_1$= 0.5 V, $U_2$= - 0.417 V
$\omega_z/2\pi$ ~ 100 MHz

P. Bushev et al., Eur. Phys. J. D (in press)

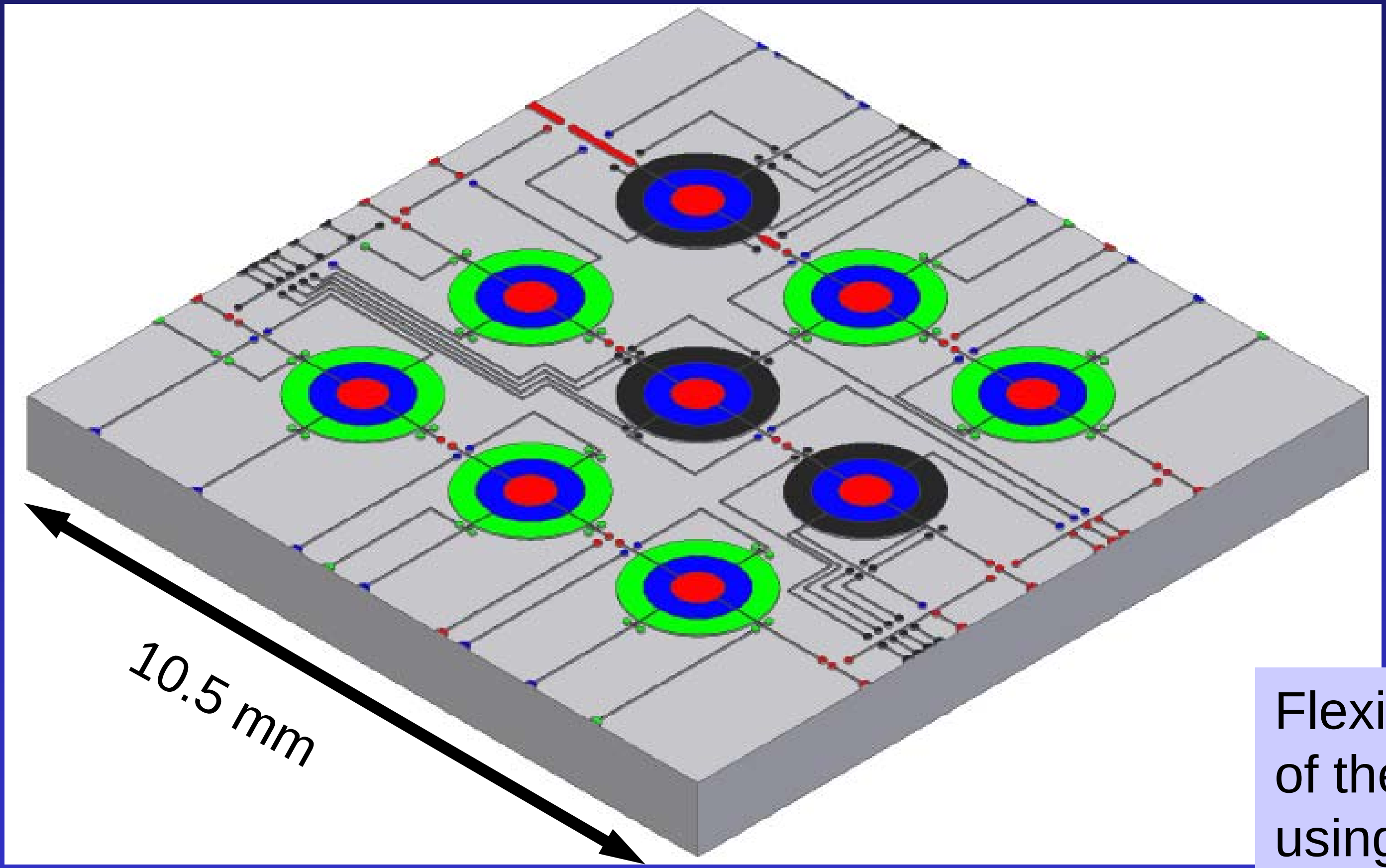
Cloud of electrons
@ T = 100 mK

# Micro fabricated trap with multiple ring electrodes




The effective electrode size can vary from $R_{0/1/2}$ = 1500/3000/4500 $\mu$m down to $R_{0/1/2}$ = 50/100/150 $\mu$m

# Planar electron micro-trap arrays

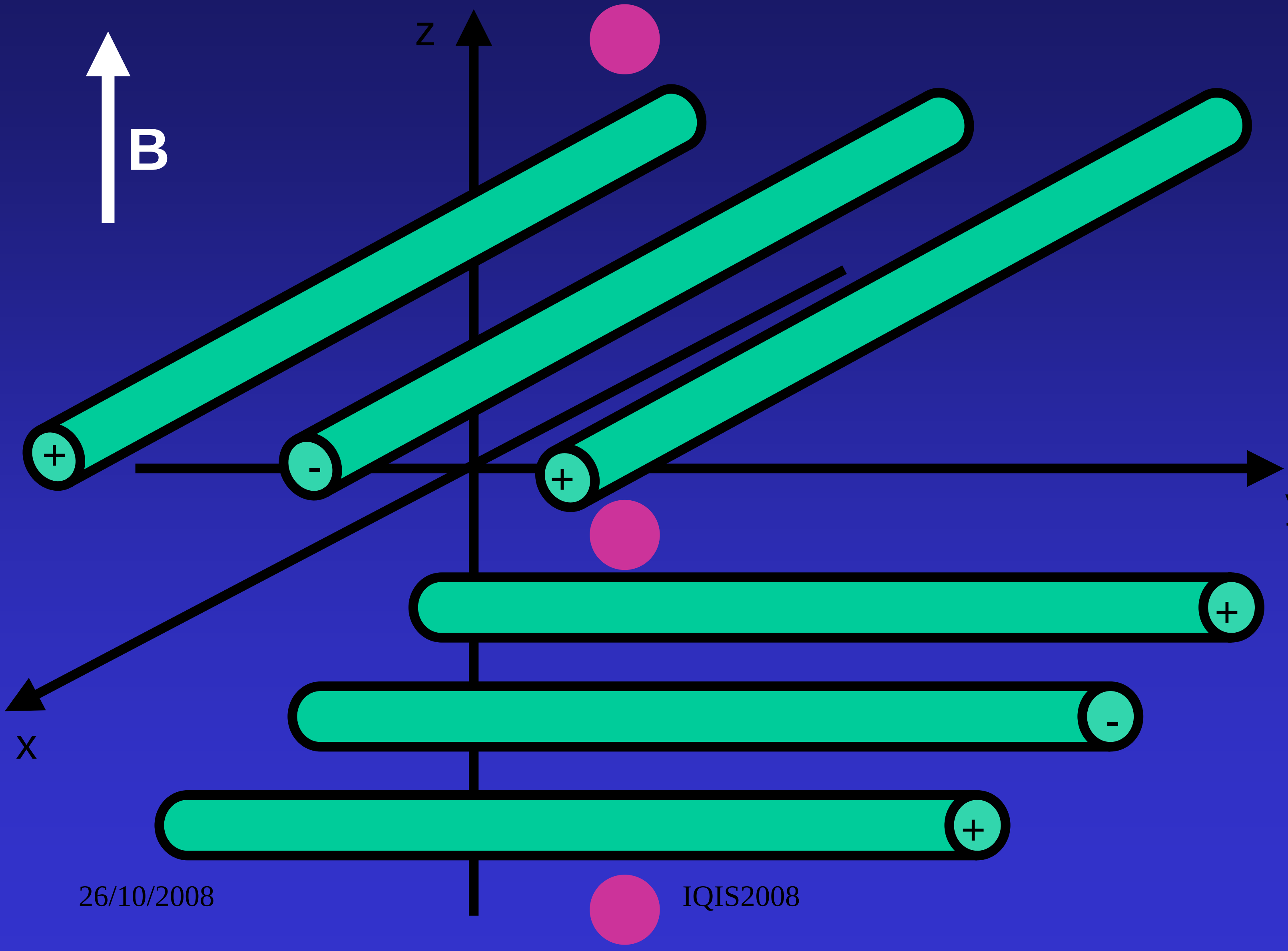



Drawing courtesy of Michael Hellwig (University of Ulm)

Interconnected micro traps with
$R_0 = 300\ \mu m$
$R_1 = 600\ \mu m$
$R_2 = 900\ \mu m$

Flexible variation of the connections using bonding pads on the gold surface

J. R. Castrejon-Pita and R. C. Thompson, Phys. Rev. A **32**, 013405 (2005)



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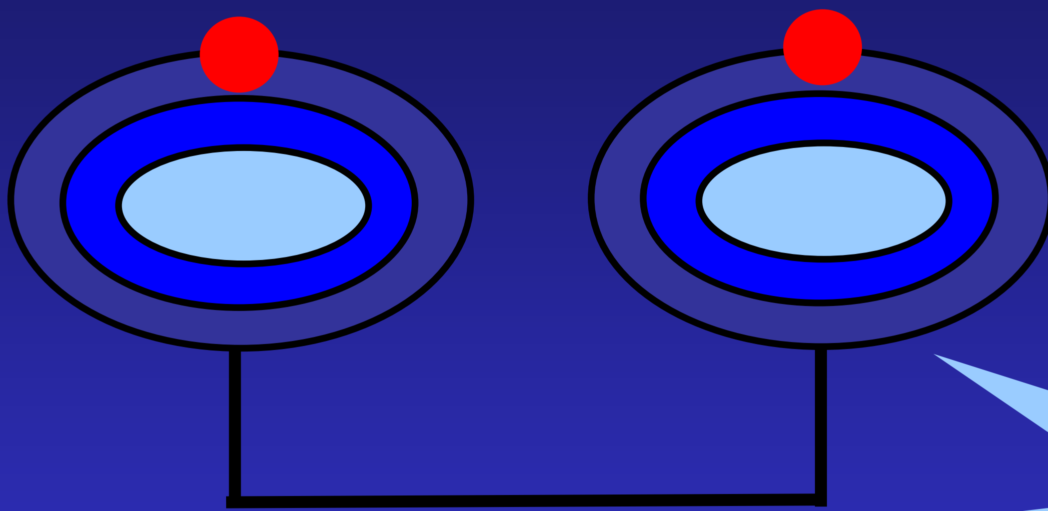
# Coupling the axial qubits with wires

J. Zurita-Sánchez and C. Henkel, PRA **73**, 063825 (2006)
J. Zurita-Sánchez and C. Henkel, New J. Phys. **10**(08), 083021 (2008)



Information exchange wire

The oscillating image charges of two electrons are coupled to each other

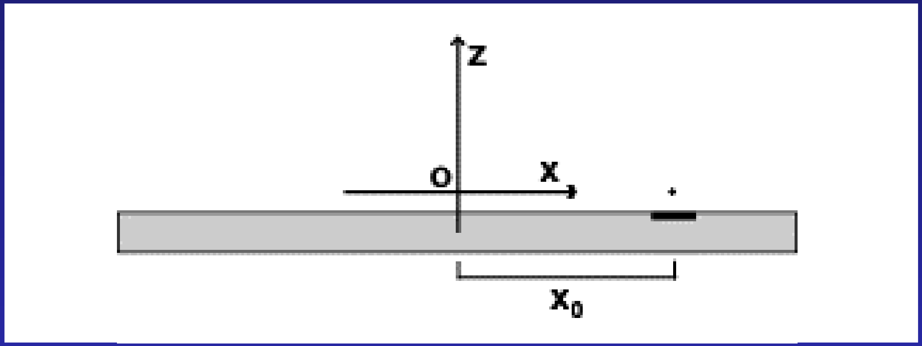
Coherent swapping of excitation between the traps

$$|10\rangle = |n_1 = 1, n_2 = 0\rangle \to |01\rangle = |n_1 = 0, n_2 = 1\rangle$$

# Linear array of traps

F. Mintert and Ch. Wunderlich, Phys. Rev. Lett. **87**, 257904 (2001)

A planar trap at a distance $x_0$ from the center of the substrate




Inhomogeneous magnetic field

(linear gradient)

$$\mathbf{B}_1 = b\left( z\mathbf{k} - \frac{x}{2}\mathbf{i} - \frac{y}{2}\mathbf{j} \right)$$

Direct Coulomb interaction

$$H_{i,j}^{C} = \frac{e^2}{4\pi\varepsilon_0 \,|\, \mathbf{r}_i - \mathbf{r}_j \,|}$$

The magnetic field gradient couples each electron internal (spin) and external (motional) degrees of freedom

$$H_i^{NC} \approx -\hbar\omega_m a_{m,i}^{+} a_{m,i} + \hbar\omega_c a_{c,i}^{+} a_{c,i} + \hbar\omega_z a_{z,i}^{+} a_{z,i} + \frac{\hbar}{2}\omega_s \sigma_i^z$$

$$+\frac{g}{4}\hbar\omega_z \varepsilon\left(a_{z,i}^{+} + a_{z,i}\right)\sigma_i^z - \frac{g}{4}\hbar\omega_z \varepsilon\sqrt{\frac{\omega_z}{\tilde{\omega}_c}}\left(\sigma_i^{(-)} a_{c,i}^{+} + \sigma_i^{(+)} a_{c,i}\right)$$

$$\varepsilon = \frac{|e|b\Delta z}{m\omega_z} = \frac{|e|b}{m\omega_z}\sqrt{\frac{\hbar}{2m\omega_z}}$$

Size of the ground state

The Coulomb interaction couples axial and cyclotron motion of different electrons

$$H_{i,j}^{C} \approx \hbar \xi_{i,j} \left( a_{z,i}^{\;+} + a_{z,i} \right) \left( a_{z,j}^{\;+} + a_{z,j} \right) - \hbar \xi_{i,j} \frac{\omega_z}{\tilde{\omega}_c} \left( a_{c,i} a_{c,j}^{\;+} + a_{c,i}^{+} a_{c,j} \right)$$

$$\xi_{i,j} = \frac{e^2}{8\pi\varepsilon_0 m \omega_z d_{i,j}^3} = \frac{1}{\hbar} \frac{e^2}{4\pi\varepsilon_0 d_{i,j}} \left( \frac{\Delta z}{d_{i,j}} \right)^2$$

Remove the coupling between internal and external degrees of freedom with a canonical transformation

$$H \rightarrow e^{S} H e^{-S}$$

$$S = \frac{g}{4}\varepsilon \sum_{i=1}^{N}\left[\sigma_i^z\left(a_{z,i}{}^{+} - a_{z,i}\right) + \frac{\omega_z}{\omega_a}\sqrt{\frac{\omega_z}{\tilde{\omega}_c}}\left(\sigma_i^{(-)}a_{c,i}{}^{+} - \sigma_i^{(+)}a_{c,i}\right)\right]$$

$$\omega_a \equiv \omega_s - \omega_c$$

anomaly frequency

# Effective spin-spin Hamiltonian

G. Ciaramicoli, I. M., and P. Tombesi, PRA **75**, 032348 (2007)

$$H_s \approx \sum_{i=1}^{N} \frac{\hbar}{2}\omega_{s,i}\sigma_i^z + \frac{\hbar}{2}\sum_{i<j}^{N}\left[2J_{i,j}^z\sigma_i^z\sigma_j^z - J_{i,j}^{xy}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right)\right]$$

$$J_{i,j}^z = \left(\frac{g}{2}\right)^2 \xi_{i,j}\varepsilon^2 = \left(\frac{g}{2}\right)^2 \frac{\hbar e^4}{16\pi\varepsilon_0 m^4}\frac{b^2}{\omega_z^4 d_{i,j}^3}$$

Dipolar decay

$$J_{i,j}^{xy} = 10^6\left(\frac{g}{4}\right)^2 \xi_{i,j}\varepsilon^2\left(\frac{\omega_z}{\omega_c}\right)^4 = 10^6\left(\frac{g}{2}\right)^2 \frac{\hbar e^4}{64\pi\varepsilon_0 m^4}\frac{b^2}{\omega_c^4 d_{i,j}^3}$$

# Array of trapped electrons as an NMR molecule

G. Ciaramicoli, F. Galve, I.M., and P. Tombesi, Phys. Rev. A **72** , 042323 (2005)
D. Mc Hugh and J. Twamley, Phys. Rev. A **71**, 012315 (2005)

$$H_s' \cong \sum_{i=1}^{N} \frac{\hbar}{2}\omega_{s0,i}\sigma_{z,i} + \sum_{i>j}^{N} \frac{\hbar}{2}\pi J_{i,j}\sigma_{z,i}\sigma_{z,j}$$

$$\omega_{s0,i} = \frac{geB_0}{2m}\left(1+\frac{b^2 x_{i,0}^{\;2}}{8B_0^{\;2}}\right)$$

the spin (qubit) frequency depends on the trap position

$$J_{i,j} = \frac{g^2}{2\pi}\xi_{i,j}\varepsilon^2 \propto \frac{b^2}{\omega_z^{\;4} d_{i,j}^{\;3}}$$

the coupling constant depends on the magnetic gradient, the axial frequency, and the inter-particle distance

## Estimate of the spin-spin coupling strength *J*

The axial frequency $\omega_z/(2\pi)$ is 100 MHz

| | $d$ = 100 μm | $d$ = 50 μm | $d$ = 10 μm |
|---|---|---|---|
| $b$ = 50 T/m | 2.3 Hz | 18 Hz | 2300 Hz |
| $b$ = 500 T/m | 0.23 kHz | 1.85 kHz | 230 kHz |

# A channel for quantum communication

M. Avellino, A. J. Fisher, and S. Bose, *Quantum Communication in Spin Systems with Long-Range Interactions,* Phys. Rev. A **74**, 012321 (2006).

$$H_s = \sum_{i=1}^{N} \frac{\hbar}{2} \omega_s \sigma_i^z - \hbar \sum_{i<j}^{N} \left[ 2 J_{i,j}^z \sigma_i^z \sigma_j^z - J_{i,j}^{xy} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \right]$$

with $J_{i,j}^{xy} = J_{i,j}^{z}$ . Transmission fidelity, up to 20 spins, larger than 90%! The transfer time scales as the cube of the chain length.

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# Spin state manipulation

## Microwave field at the spin transition frequency

$$\mathbf{b}_p(t) = b_p\left[\cos(\omega t+\theta)\hat{x} + \sin(\omega t+\theta)\hat{y}\right]$$

$$H_{\text{int}} = -\boldsymbol{\mu}\cdot\mathbf{b}(t) = \frac{g}{2}\mu_B\boldsymbol{\sigma}\cdot\mathbf{b}_p(t) = \frac{\hbar\Omega}{2}\left[\sigma^{(+)}\,\mathrm{e}^{-i(\omega t+\theta)} + \sigma^{(-)}\,\mathrm{e}^{i(\omega t+\theta)}\right]$$

$$\left|\downarrow\right\rangle \rightarrow \cos\left(\frac{\Omega t}{2}\right)\left|\downarrow\right\rangle - i\,\mathrm{e}^{-i\theta}\sin\left(\frac{\Omega t}{2}\right)\left|\uparrow\right\rangle$$

$$\left|\uparrow\right\rangle \rightarrow \cos\left(\frac{\Omega t}{2}\right)\left|\uparrow\right\rangle - i\,\mathrm{e}^{i\theta}\sin\left(\frac{\Omega t}{2}\right)\left|\downarrow\right\rangle$$

# Single qubit operations

Spin flip with a π pulse
and θ = 0

$$F \equiv -i \bigotimes_{j=1}^{N} \left( \sigma_j^{(+)} + \sigma_j^{(-)} \right)$$

Pseudo-Hadamard
with a π/2 pulse

$$G_x \equiv \bigotimes_{j=1}^{N} \frac{\left(1 - i\sigma_j^x\right)}{\sqrt{2}} \quad \text{for} \quad \theta = 0$$

$$G_x^+ \equiv \bigotimes_{j=1}^{N} \frac{\left(1 + i\sigma_j^x\right)}{\sqrt{2}} \quad \text{for} \quad \theta = \pi$$

$$G_y \equiv \bigotimes_{j=1}^{N} \frac{\left(1 - i\sigma_j^y\right)}{\sqrt{2}} \quad \text{for} \quad \theta = \pi/2$$

$$G_y^+ \equiv \bigotimes_{j=1}^{N} \frac{\left(1 + i\sigma_j^y\right)}{\sqrt{2}} \quad \text{for} \quad \theta = -\pi/2$$

# Engineering the spin Hamiltonian

G. Ciaramicoli, I. M., and P. Tombesi, PRA **78**, 012338 (2008)

Goals:

- Tuning of the effective magnetic field
- Design and control of the spin-spin coupling
- Change the interaction range and topology

How?

With sequences of resonant microwave pulses alternated with periods of free evolution

# Reducing the effective magnetic field

$$H_s = H_0 + H_c$$

$$H_0 \equiv \frac{\omega_s}{2} \sum_{i=1}^{N} \sigma_i^z$$

$$H_c \equiv \frac{1}{2} \sum_{i<j}^{N} \left[ 2 J_{i,j}^z \sigma_i^z \sigma_j^z - J_{i,j}^{xy} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \right]$$

Spin flips alternated with free evolution

$$F^{-1} e^{-iH_s t_2} F e^{-iH_s t_1} = \exp[-iH_{\text{eff}}(t_1 + t_2)]$$

$$H_{\text{eff}} \equiv \frac{t_1 - t_2}{t_1 + t_2} H_0 + H_c$$

# Design of the XY model

Starting from

$$H_c \approx \sum_{i<j}^{N} J_{i,j}^{z} \sigma_i^{z} \sigma_j^{z}$$

$$G_x e^{-iH_c t_1} G_x^{+} = \exp\left( -i \sum_{j<k}^{N} J_{j,k}^{z} \sigma_j^{y} \sigma_k^{y} t_1 \right)$$

$$G_y e^{-iH_c t_2} G_y^{+} = \exp\left( -i \sum_{j<k}^{N} J_{j,k}^{z} \sigma_j^{x} \sigma_k^{x} t_2 \right)$$

We obtain

$$H_{\text{eff}} = \frac{t_2}{t_1 + t_2} \sum_{i<j}^{N} J_{i,j}^{z} \sigma_i^{x} \sigma_j^{x} + \frac{t_1}{t_1 + t_2} \sum_{i<j}^{N} J_{i,j}^{z} \sigma_i^{y} \sigma_j^{y}$$

# Approximation of a unitary operator

A.T. Sornborger and E.D. Stewart, PRA **60**, 1956 (1999)

Target unitary operator *U* = exp(-*iHt* ) with $H = \sum_{i=1}^{n} \tau_i A_i$

$$t = \sum_{i=1}^{n} t_i \quad \text{and} \quad \tau_i = t_i / t$$

$$S = \overline{S}_1 S_1 \overline{S}_1 S_{-2} \overline{S}_1 \overline{S}_1 \overline{S}_1 \overline{S}_1 S_1 \overline{S}_1 S_1 S_1 S_1 S_1 \overline{S}_{-2} S_1 \overline{S}_1 S_1$$

$$S_k = e^{-i\frac{k}{12} A_1 t_1} e^{-i\frac{k}{12} A_2 t_2} \cdots e^{-i\frac{k}{12} A_n t_n}$$

$$\overline{S}_k = e^{-i\frac{k}{12} A_n t_n} \cdots e^{-i\frac{k}{12} A_2 t_2} e^{-i\frac{k}{12} A_1 t_1}$$

The sequence *S* approximates *U* to the fourth order in *t*

# Fidelity

The error is the distance between the target unitary operator *U* and our approximation *U’*

$$E \equiv \|U - U'\| \equiv \max_{|\varphi\rangle : \||\varphi\rangle\| = 1} \left| (U - U') |\varphi\rangle \right|$$

For a sequence *S* producing the effective evolution for a time *t*

$$E_S \leq (J^z t)^5 f(N) \quad \text{with} \quad f(N) \ \text{linear function}$$

For *m* iterations of *S*, giving a total evolution time *T* = *mt*

$$E \leq \frac{(J^z T)^5 f(N)}{m^4}, \quad \text{since} \quad E \leq m E_S$$

# Summary & future perspectives

- Scalable planar Penning traps of variable size and geometry;
- Design of an effective spin-spin interaction;
- Universal quantum gates with a single trapped electron;
- Short distance quantum communication (Giulia Gualdi’s poster today evening);
- Transmission of a qubit state in a quantum processor (instead of using a swapping gate);
- Entanglement of distant qubits;
- Simulation of quantum systems with Ising, XY, or XYZ spin-spin interaction;
- Observation of quantum phase transitions (?!?)

6th FP – Information Society Technologies contract no. FP6-003772

# Quantum Computing with Trapped Electrons

http://fisica.unicam.it/quele/

QUELE

- Università di Camerino
- Johannes Gutenberg Universität Mainz
- Universität Ulm
- Politechnika Poznanska
- Ernst-Moritz-Arndt-Universität Greifswald
- Czech Technical University in Prague
- Universität Potsdam