Transition features in quantum communication channels with correlated noise

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OUTLINE

Optimise the performance of quantum communication channels in the presence of correlated noise

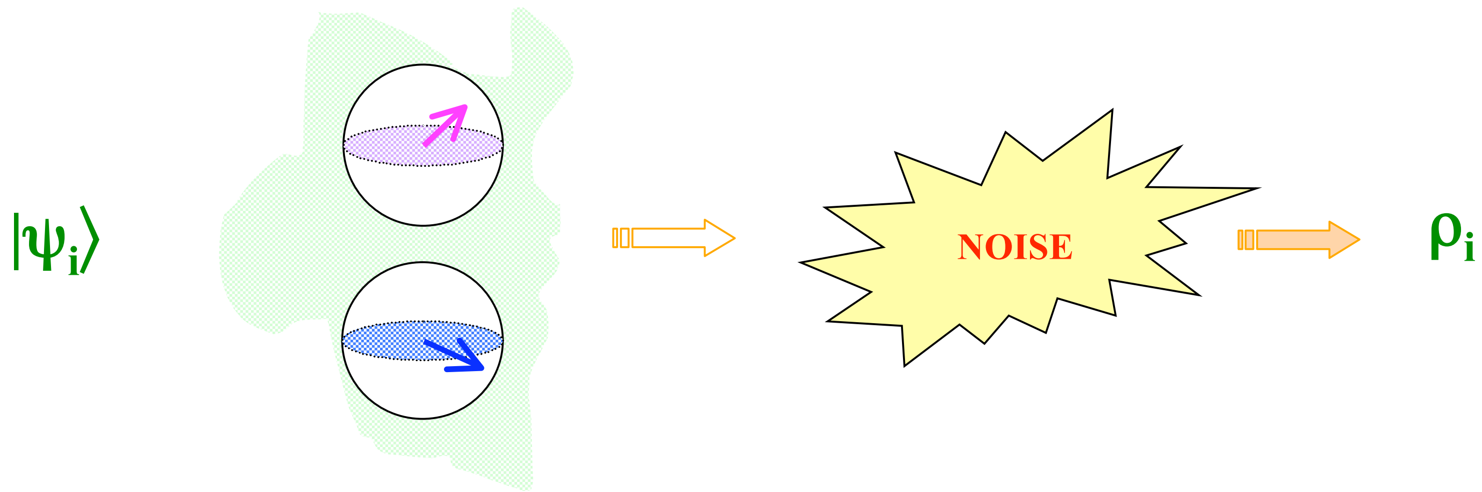
Depolarising channel with Pauli correlations

Depolarising channel with shift correlations

Transition in the input entanglement depends on the form of the correlation

SCENARIO

Information encoded into quantum states of two qubits



Mutual information: $I(\varepsilon) = S(\rho_{av}) - \Sigma_i\, p_i\, S(\rho_i)$

$\varepsilon = \{ p_i , \psi_i \}, \quad \rho_{av} = \Sigma_i\, p_i\, \rho_i$

$S(\rho) = -Tr[\rho \log \rho]$

Von Neumann entropy

Capacity: $C = \sup_{\varepsilon} I(\varepsilon)$

Maximal output purity: $\sup\_ [Tr(\rho^2)]^{1/2}$

$\sup\_ [Tr(\rho^p)]^{1/p}$

Channels with correlation

$$\Phi = (1 - ì\ )\varphi \otimes \varphi + ì\ \Gamma_{corr}$$

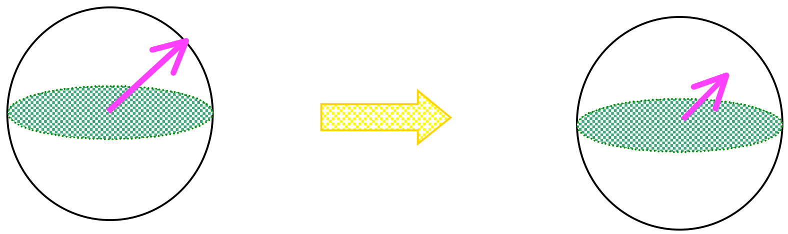
_: qubit depolarising channel

$\pi = (I + \Sigma_j\, s_j\ \sigma_j\,)\,/2$ ⇒ $\_(\pi) = (I + \_\ \Sigma_j\, s_j\ \sigma_j\,)\,/2$

$= \_\ \pi + (1\text{-}\_)\,/2\ \ I$

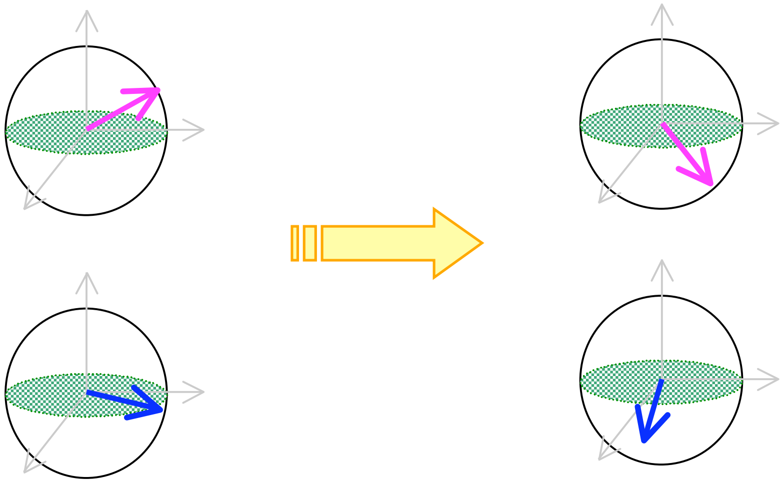
With prob (1+3_)/4: state unchanged

With prob (1-_)/4: apply σ_x, σ_y or σ_z



For _=0 product states maximise I and OP

Pauli correlation



With probability (1+3_)/4 : state unchanged

With probability (1-_)/4 : apply σ_x, σ_y or σ_z on both qubits

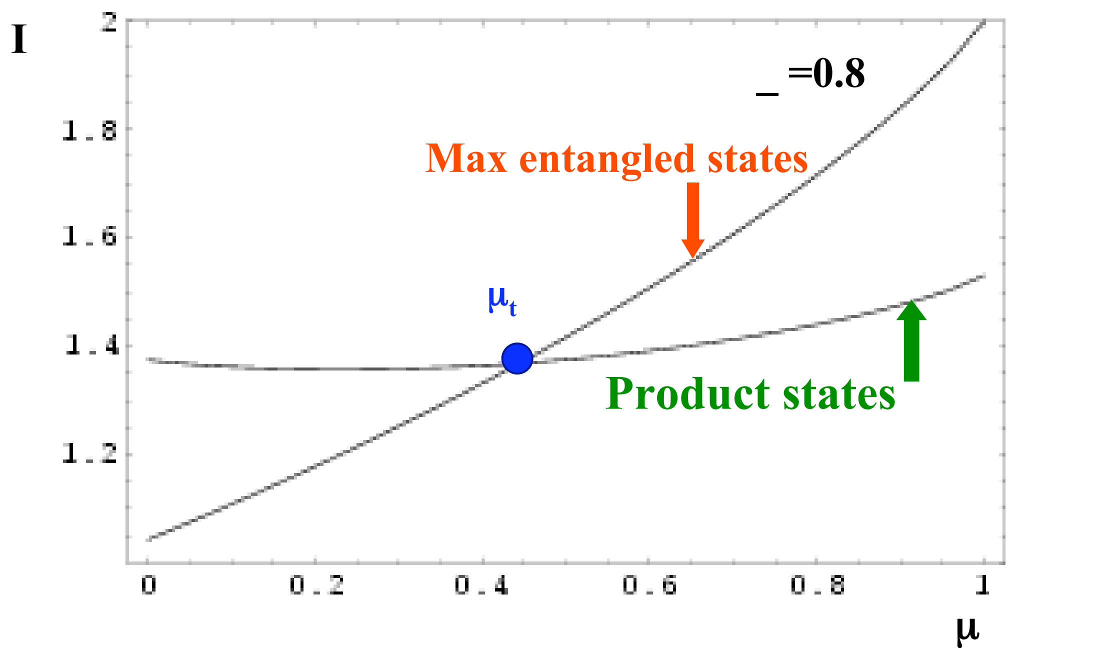
Bell states are left unchanged, and maximise I and the output purity

Transition in input entanglement

Threshold value μ_t=_/(1+_) such that:

$\mu < \mu_t$ I and OP max for product states

$\mu > \mu_t$ I and OP max for max entangled states



(C.M. & M. Palma, PRA02)

Generalisations of Pauli correlation

Quasi-classical depolarising channel

$\_{z} = \_ , \quad \_{x} = \_{y} = 0$

(C.M., M. Palma & S. Virmani, PRA04)

Non isotropic Pauli channel **(D. Daems, PRA07)**

Some generalised Pauli channels in dimension d **(Karpov, Daems & Cerf, PRA06)**

Same transition behaviour!

Shift correlation

corr : |⟩ ⟨_| |_⟩ = (|00⟩ + |11⟩)/2

$$\Phi_{\mu,\lambda}(R) = (1 - ì\ )\varphi_{\lambda} \otimes \varphi_{\lambda}(R) + ì\mathrm{Tr}[R]\,|\beta\rangle\langle\beta|$$

Non covariant, non unital

Correlation term independent of the input state

Optimisation

For p=2 optimisation leads to the parametrisation of the input:

|_⟩ = cos(_/2) |00⟩ + sin(_/2) |11⟩

Threshold for _:

μ_t=(1-_^{2})/(2-_^{2})

For _< μ_t optimal input states are entangled ($U^T \otimes U$ |_⟩) with

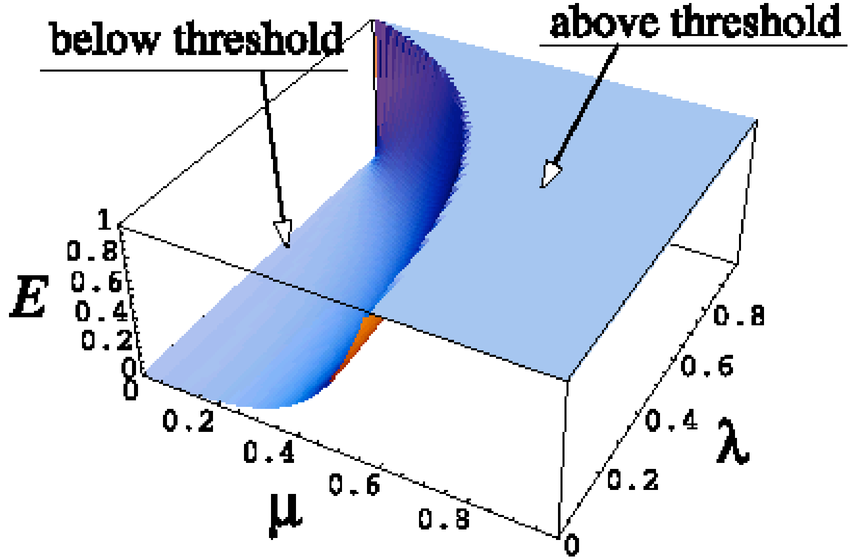
_ = arcsin [_/(1- _)(1-_^{2})]

For _> μ_t optimal input state is |_⟩ (max entangled)

Product states are never optimal for _>0 !

No transition!

Optimal input entanglement



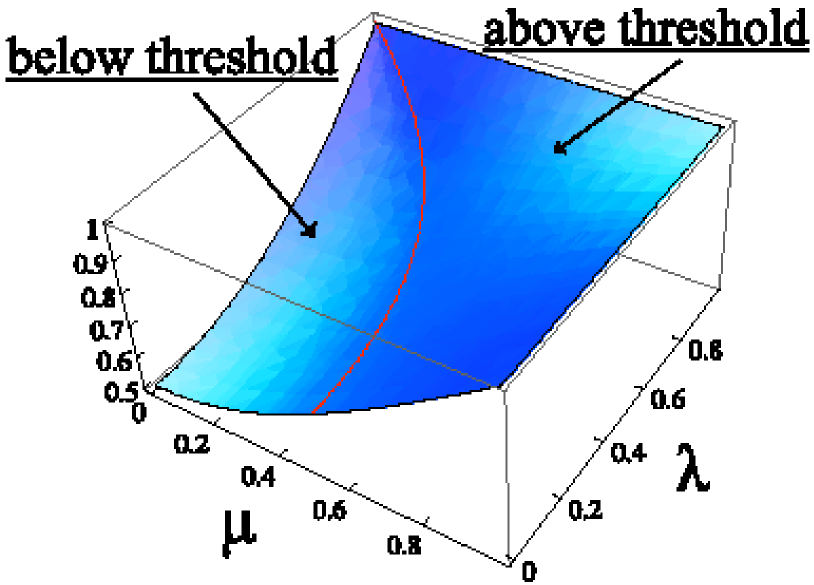
Single qubit: _$_{1}$= (I + cos_ σ_z)/2

E = h(cos _) h(x)=-(1+x)/2 ln[(1+x)/2]-(1-x)/2 ln[(1-x)/2]

Monotone function of _, E=0 for _=0 and E=1 for _> μ_t

Maximal output purity

p=2



Numerical results support the conjecture that the same optimisation holds also for p>1 and for the minimal output entropy

(F. Caruso, V. Giovannetti, C.M. & MB Ruskai, PRA08)

Comment

Different behaviour from depolarising channel with shift in dimension 4:

$$\Phi_{\mu,\lambda}(R) = (1-\mu)[\lambda R + (1-\lambda)I_4/4] + \mu\,|\beta\rangle\langle\beta|$$

Always optimised by the shift state $|\_\rangle$!

SUMMARY

Analysis of two-qubit depolarising channel with correlated noise

The channel performance depends on the form of the correlation

Pauli correlations: transition behaviour in the input entanglement in the optimisation of I and OP

Shift correlations:

channel always optimised by entangled states, no transition!